Importance Sampling

Variance reduction would be critical when pricing options that are deep out of money. For example, if a put option is deep out of money, which means S0 is much greater than K, then the most of the payoffs generated by the simulation would be zero, thus leading to a high pricing standard error. In order to generate more efficient simulation paths, the main idea of importance sampling is to find a form for the Radon-Nikodym derivative that can effectively change the probability measure.

[] gives the form of the exponential twisting,





whereare the independent process generated in the difference-of-gammas with new parameters.

So the option price would be



Therefore,



Here we try to price the deep-out-of money European put with S0=100, and K=45. For the selection of new parameters, we try to decrease the original, and increase the original. The parameters are as follows,

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| 0.25 | -0.05 | 0.5 | 0.35 | -0.1 | 0.5 |

And the pricing result is in table\_\_\_

|  |  |  |
| --- | --- | --- |
|  | Estimated price | Standard error |
| VG Diff Gamma | 0.025807280 | 0.001583751 |
| VG Importance Sampling | 0.0237484936 | 0.0005620372 |

As we can see in the table, there is indeed some variance reduction for the price, but is not very significant. The reason might be the selection of new probability measure. We are still working on how to make an efficient selection of new measure parameters.